E_9 symmetry in the Heterotic String on S^1 and the Weak Gravity Conjecture

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Based on `arXiv:2203.01341` with A. Herraez, M. Graña

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Motivation

- Extend the scan of symmetry enhancements in the Heterotic theory on S^1 to infinite distance in moduli space. [Fraiman, Graña, Nuñez '18]

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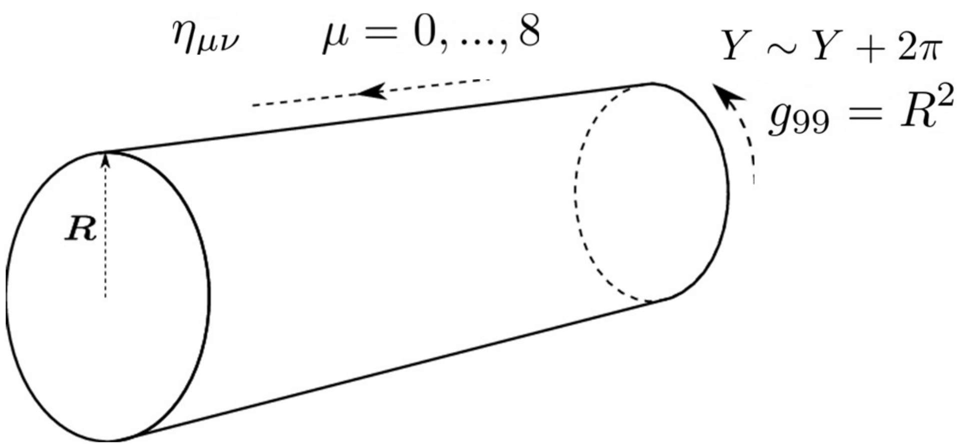
- How do these things fit together?

Heterotic string on S^1



Moduli: R and $A^{\hat{I}}$, $\hat{I} = 1, ..., 16$

Charges: $Z = (w, n, \pi^{\hat{I}})$

$\pi^{\hat{I}} \in \Gamma_8 \times \Gamma_8$

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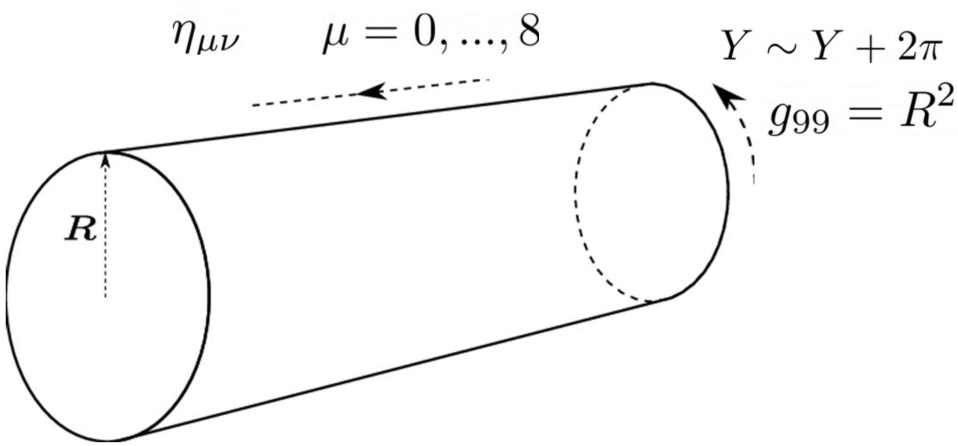

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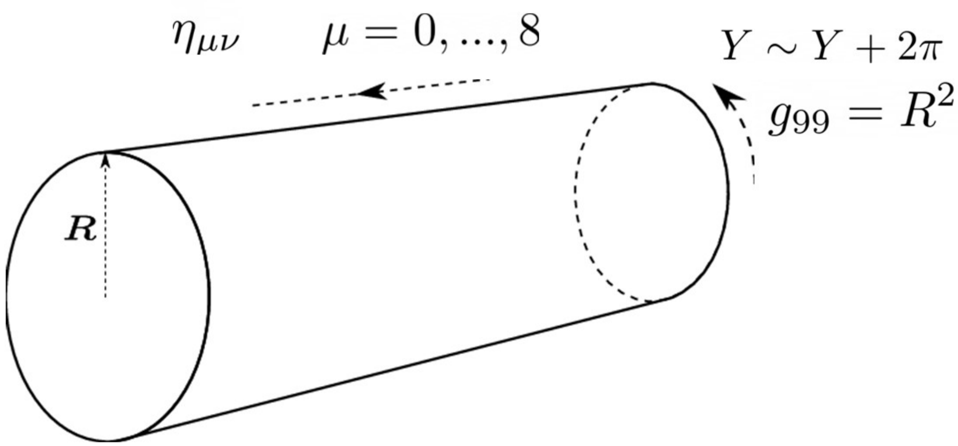

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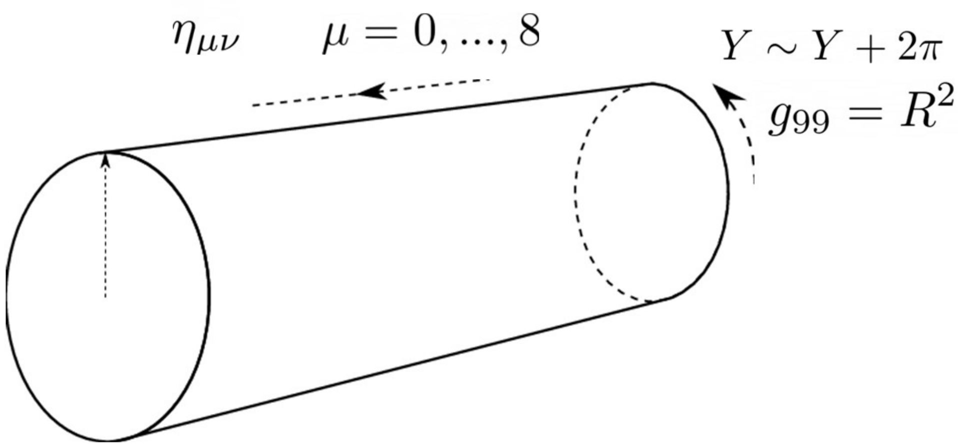
Narain Lattice
even and self dual

Heterotic string on S¹



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Spectrum: $$\begin{cases} M^2 = \boldsymbol{p}_L^2 + p_R^2 + 2\left(N + \bar{N} - \frac{3}{2}\right) \\ \boldsymbol{p}_L^2 - p_R^2 + 2\left(N - \bar{N} - \frac{1}{2}\right) = 0 \end{cases}$$

Invariant under T duality $O(1, 17, \mathbb{Z})$

Symmetry enhancements

To read the gauge algebra we must look for **massless vectors**.

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$$U(1)_R \quad \times \quad U(1)_L^{17}$$

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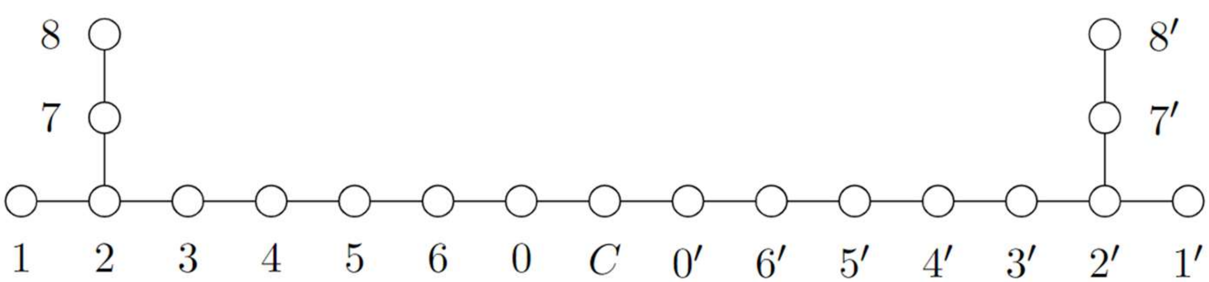

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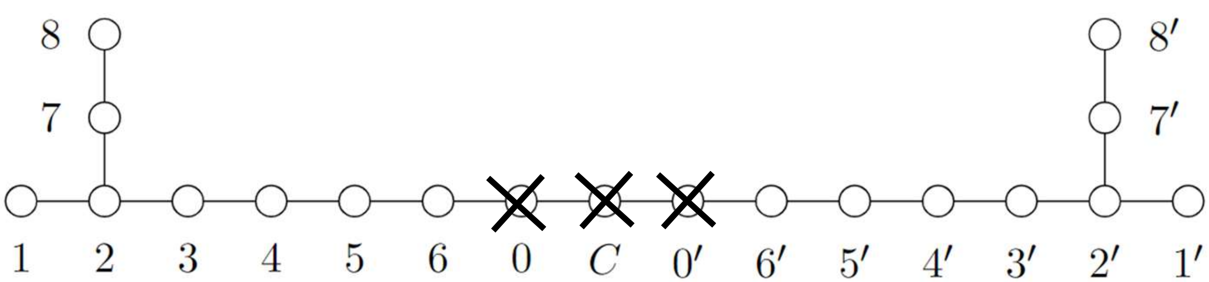

$$A = (0_8, 0_8) \quad R \neq 1$$

$$G_L^{16} \times U(1) = E_8 \times E_8 \times U(1)$$

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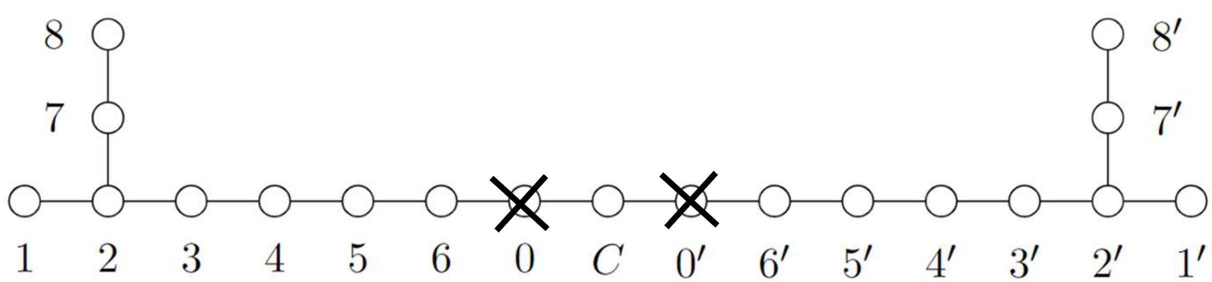

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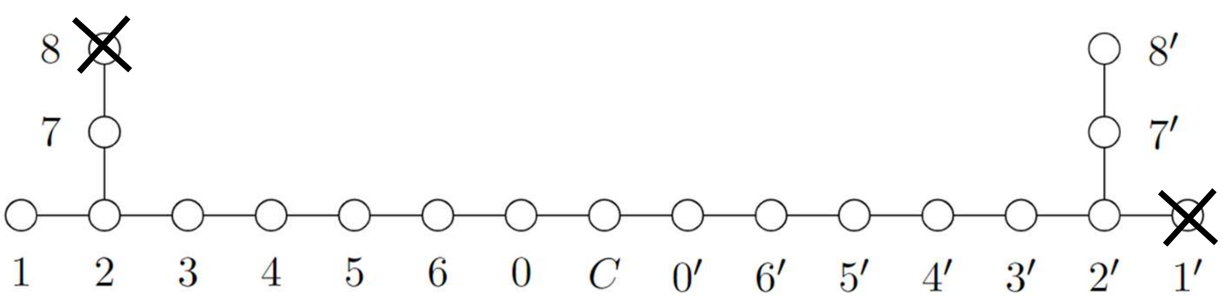

$$A = \left(0_7, -1, \frac{1}{6}, \left(-\frac{1}{6}\right)_6, \frac{5}{6}\right) \quad R^2 = \frac{1}{18}$$

$$G_L^{17} = SO(34)$$

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$\alpha^{\hat{I}}_{-1}\bar{\psi}^{\mu}_{-\frac{1}{2}}\,\lvert 0\rangle_{NS}$		$J^{\hat{I}}(z) = i\partial X^{\hat{I}}(z)$
$\alpha^{9}_{-1}\bar{\psi}^{\mu}_{-\frac{1}{2}}\,\lvert 0\rangle_{NS}$	$\longrightarrow$ VERTEX OPERATOR $\longrightarrow$	$J^{9}(z) = i\sqrt{2}\partial Y(z)$
$\bar{\psi}^{\mu}_{-\frac{1}{2}}\,\lvert 0, \pi_\alpha\rangle_{NS}$		$J^{\alpha}(z) = c_\alpha e^{i\pi^{\hat{I}}_{\alpha}X^{\hat{I}}(z)}$

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The zero modes are defined as

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They satisfy the $E_8 \times E_8 \times U(1)$ commutation relations

$$[J^{\hat{I}}, J^{\hat{J}}] = 0\,,$$
$$[J^{\hat{I}}, J^{\alpha}] = \pi^{\hat{I}}_{\alpha} J^{\alpha}\,,$$
$$[J^{\alpha}, J^{\beta}] = \begin{cases} \epsilon(\alpha,\beta)J^{\alpha+\beta} & \alpha+\beta\,\text{root}, \\ \pi^{\hat{I}}_{\alpha}J^{\hat{I}} & \alpha = -\beta\,, \\ 0 & \text{otherwise} \end{cases}$$

$$[J^9, J^9] = 0\,,$$
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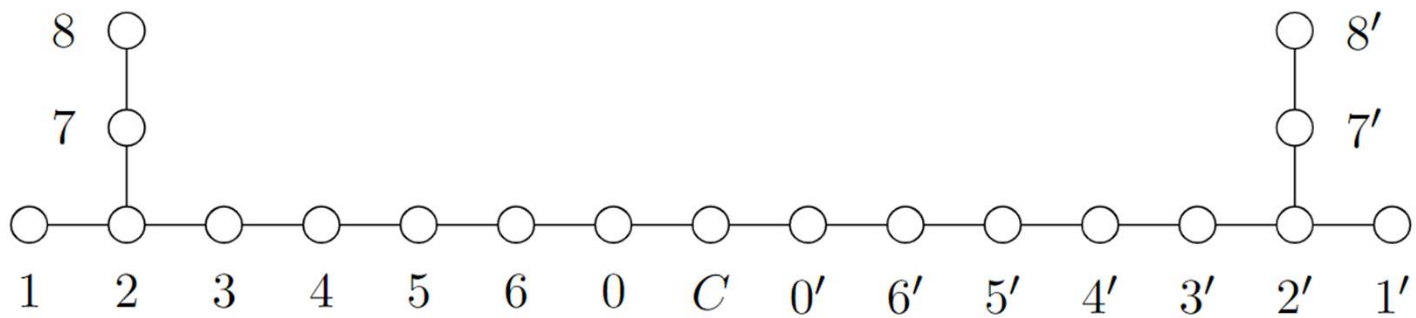
$$[J^9, J^{\alpha}] = 0\,.$$

The $E_9 \oplus E_9$ /~ algebra



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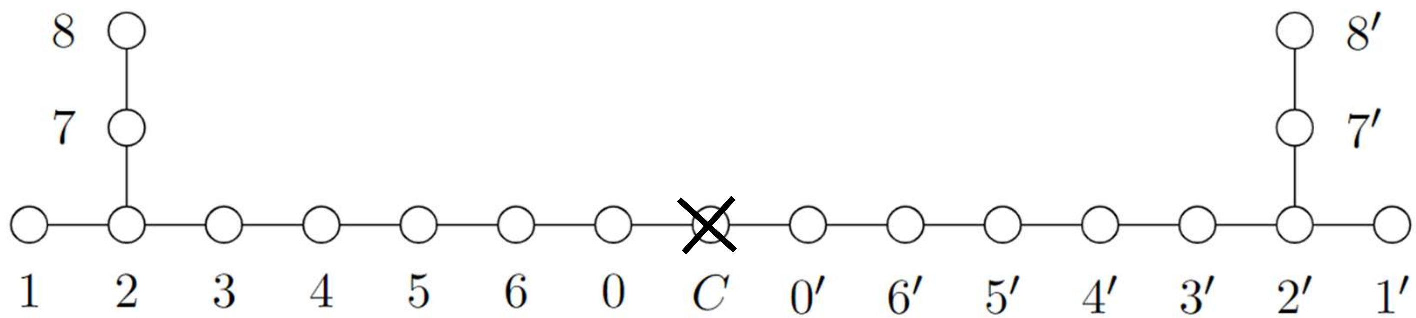

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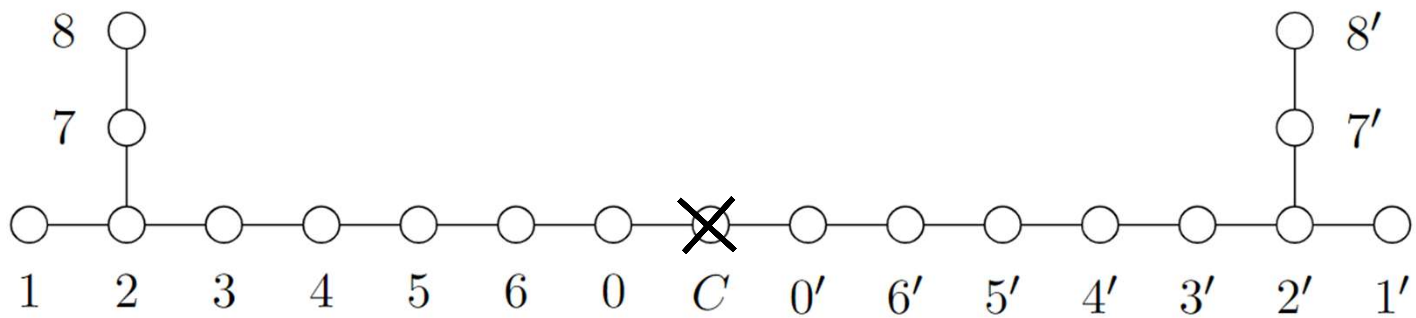

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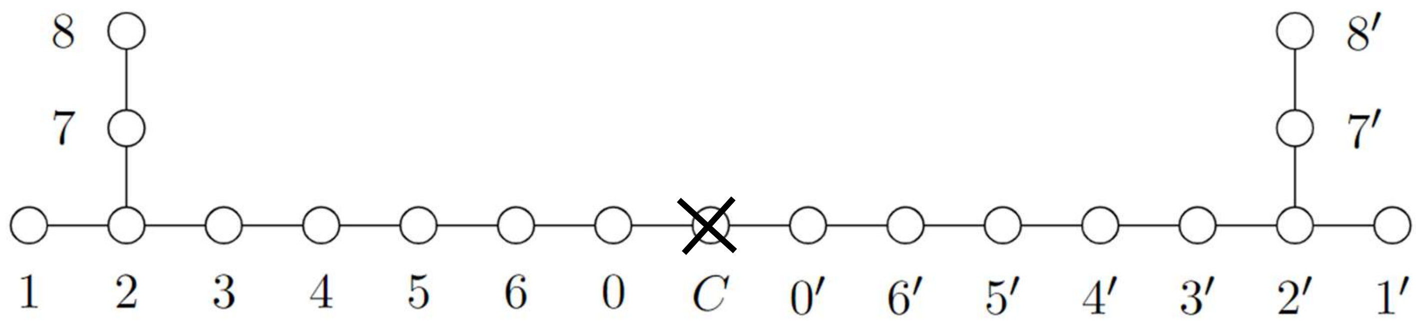

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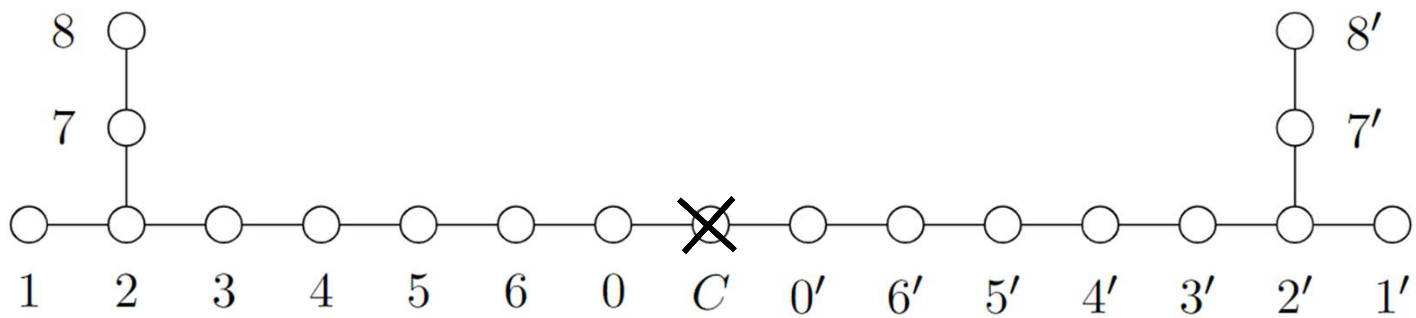

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$$\alpha^{\hat{I}}_{-1}\bar{\psi}^{\mu}_{-\frac{1}{2}}\,|0,n\rangle_{NS}\ , \qquad \bar{\psi}^{\mu}_{-\frac{1}{2}}\,|0,n,\pi_\alpha\rangle_{NS}$$

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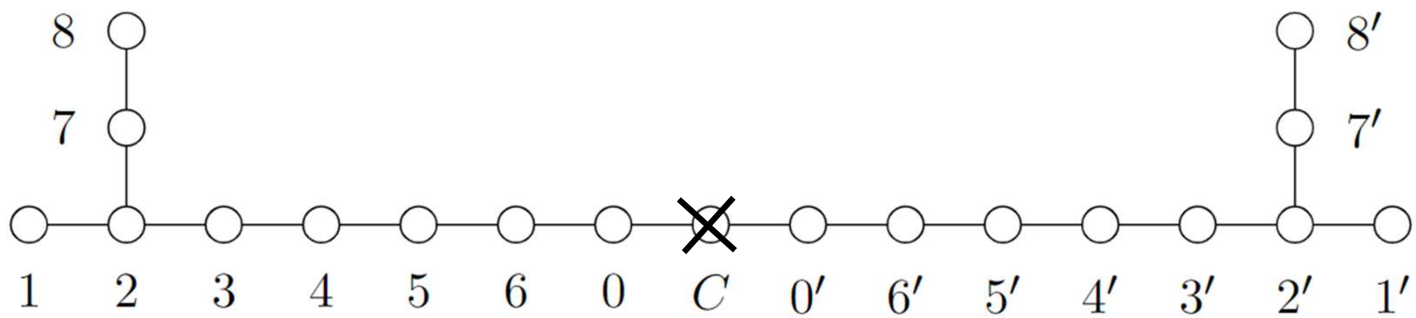
The $E_9 \oplus E_9 /\sim$ algebra



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The imaginary root $\delta = (0, 1, 0_8, 0_8)$ is shared between the two copies

$$E_9 \oplus E_9 / \sim$$

The $E_9 \oplus E_9$ /~ algebra

- Asymptotically conserved holomorphic currents (zero Wilson line) $a = \{\hat{I}, \alpha\}$

$$J^a_n(z) \equiv J^a(z) e^{inY(z)} \qquad \begin{cases} J^{\hat{I}}(z) = i\partial X^{\hat{I}}(z) \\ J^{\alpha}(z) = c_\alpha e^{i\pi^{\hat{I}}_{\alpha} X^{\hat{I}}(z)} \end{cases}$$

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central extension

- In the case of finite Wilson line redefine $X^{\hat{I}}(z) \to X'^{\hat{I}}(z) = X^{\hat{I}}(z) - A^{\hat{I}} Y(z)$

  The currents obey the same algebra.

The $E_9 \oplus E_9$ /~ algebra

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central extension

- In the $SO(32)$ theory the computation is the same, with different roots

$\Rightarrow \hat{SO}(32)$ for $R \to \infty$ and finite A $\quad (0, n, \pi^\alpha), \quad n \in \mathbb{Z},\ \pi^\alpha \in \Gamma_{16},\ |\pi|^2 = 2$

Decompactification limits

- E_8xE_8 theory in 9d
  - $R \to \infty$ and finite A: 10d E_8xE_8 ($E_9 \oplus E_9 / \sim$)

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E_9, the Weak Gravity and the Repulsive Force Conjectures

[Heidenreich, Reece, Rudelius , '16 and '20]

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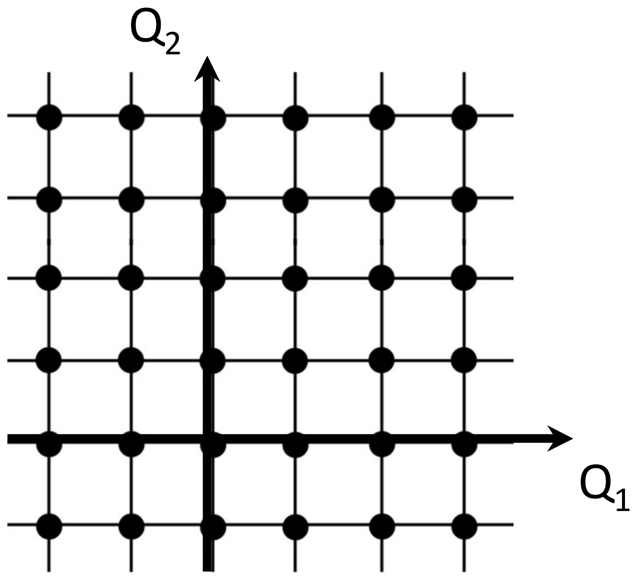

For every point in the charge lattice

LWGC superextremal state $\frac{Q_i}{M} \geq \left(\frac{Q_i}{M}\right)_{ext}$

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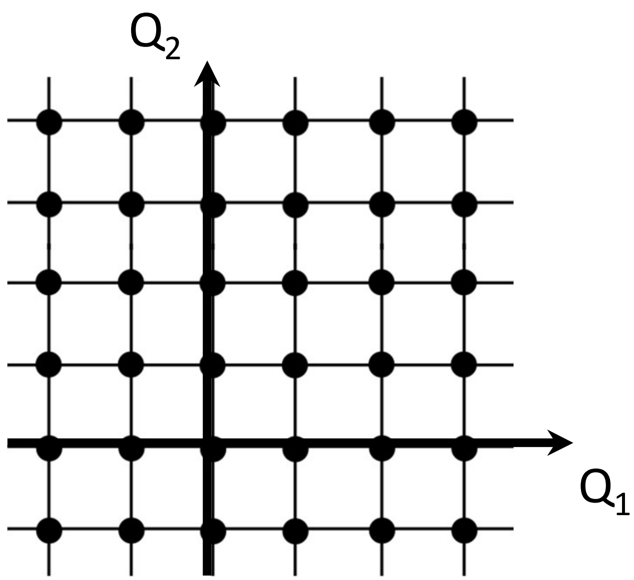

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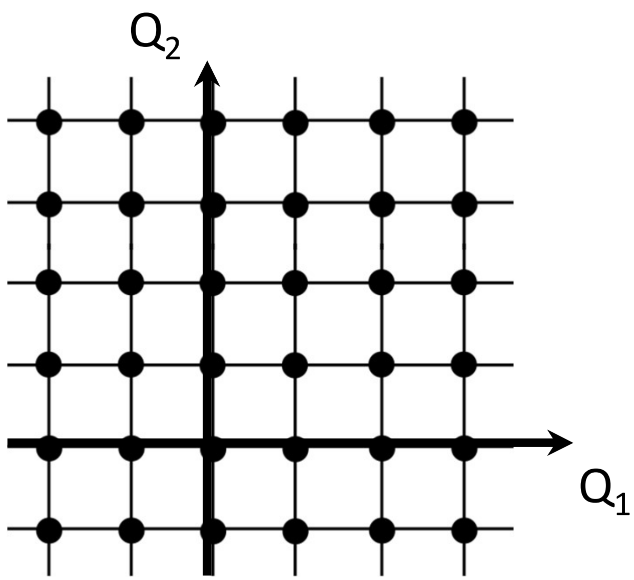

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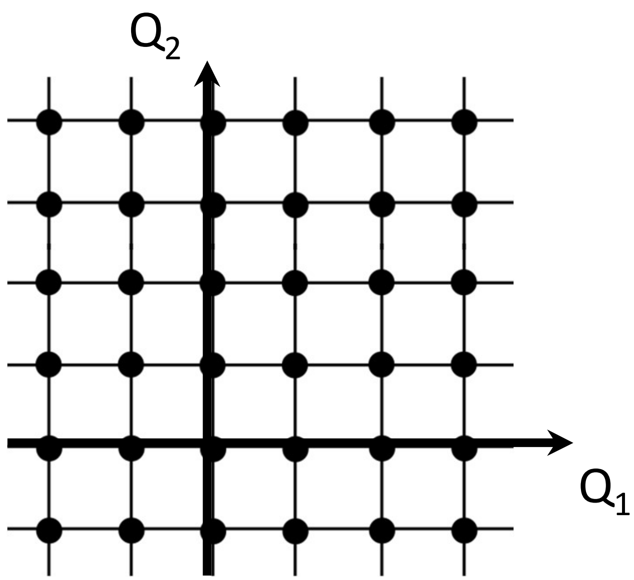

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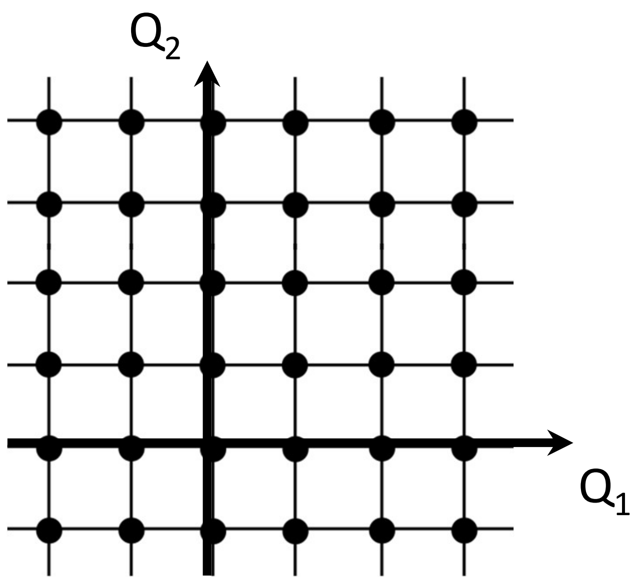

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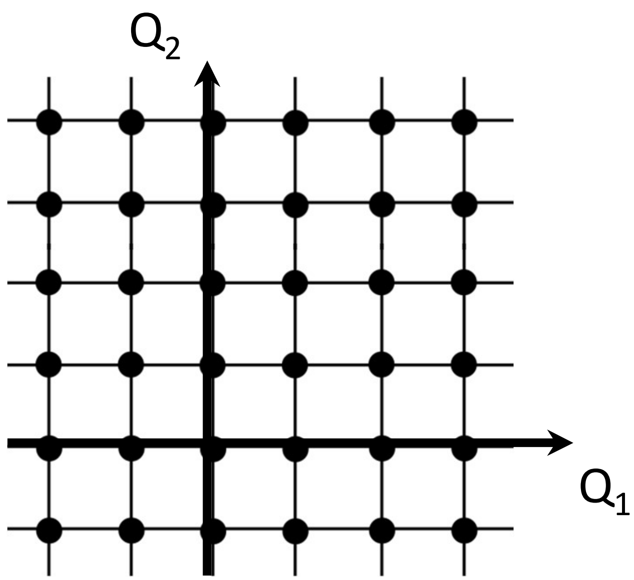

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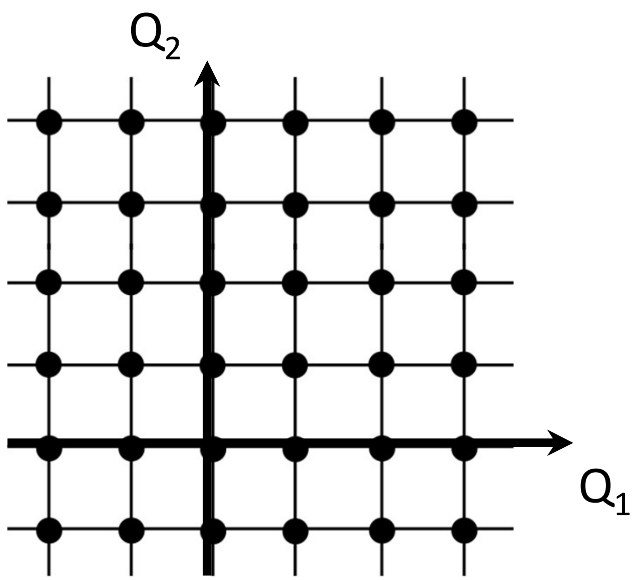

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- The WGC and RFC predict an infinite number of massless states at $R \to \infty$.

Conclusions

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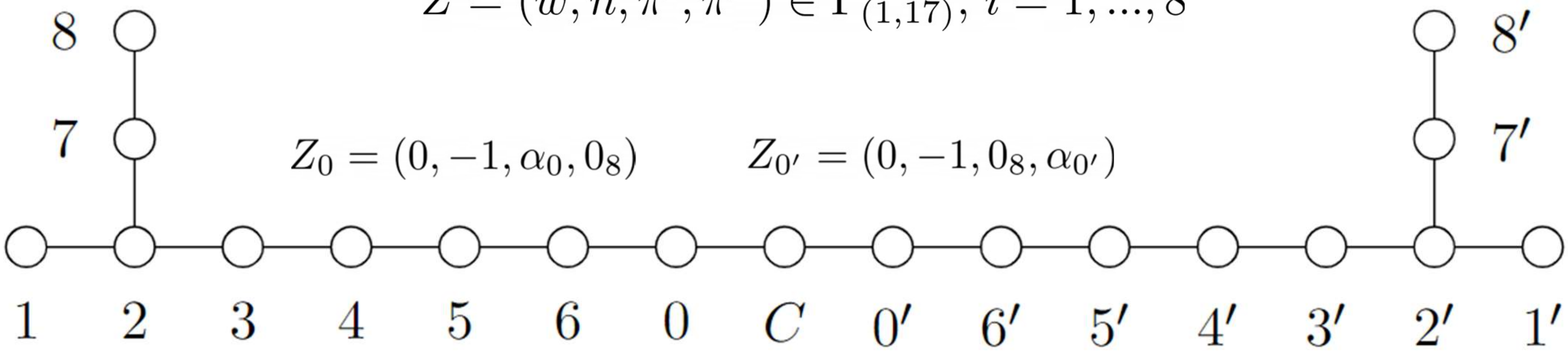
- Work in progress [VC, Grana, Herraez, Parra de Freitas, to appear] : this extends to lower dimensional Heterotic theory (and F theory dual). [Lee, Lerche, Weigand, '21]

Thank you!

Extended Dynkin Diagram

It contains information on all the possible enhancements and on the point of moduli space where they appear.

$$Z = (w, n, \pi^{i}, \pi^{i'}) \in \Gamma_{(1,17)},\ i = 1, \ldots, 8$$



$$Z_0 = (0, -1, \alpha_0, 0_8) \qquad Z_{0'} = (0, -1, 0_8, \alpha_{0'})$$

$$Z_i = (0, 0, \alpha_i, 0_8) \qquad Z_C = (1, 1, 0_8, 0_8) \qquad Z_{i'} = (0, 0, 0_8, \alpha_{i'})$$

$$U(1)_L^{17} \times U(1)_R \to G_L^r \times U(1)^{17-r} \times U(1)_R$$

$$\mathcal{A}^{\hat{I}} \equiv \frac{A^{\hat{I}}}{R}$$

- Delete $(19 - r) \geq 2$ nodes to get the Dynkin Diagram of G_L^r .
- To get the moduli, verify the equalities of the nodes that remain.

Node	Boundary of fundamental region
i	$\mathcal{A} \cdot (\alpha_i, 0_8) = 0$
0	$\mathcal{A} \cdot (\alpha_0, 0_8) = -\frac{1}{R}$
C	$R\left(\frac{1}{R^2} - \frac{\lvert\mathcal{A}\rvert^2}{2}\right) = R$
0’	$\mathcal{A} \cdot (0_8, \alpha_{0'}) = -\frac{1}{R}$
i'	$\mathcal{A} \cdot (0_8, \alpha_{i'}) = 0$

The $E_9 \oplus E_9 /\sim$ algebra

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$$J_n^a(z) \equiv J^a(z) e^{inY(z)} \qquad \begin{cases} J^{\hat{I}}(z) = i\partial X^{\hat{I}}(z) \\ J^{\alpha}(z) = c_\alpha e^{i\pi_\alpha^{\hat{I}} X^{\hat{I}}(z)} \end{cases}$$

with $\quad Y(z,\bar{z})Y(w,\bar{w}) \sim -\frac{1}{2R^2}\log|z-w|^2\,,\ X^{\hat{I}}(z)X^{\hat{J}}(w) \sim -\delta^{\hat{I}\hat{J}}\log(z-w)\,.$

- The algebra of the zero modes $(J_n^a)_0 \equiv J_n^a = \oint \frac{dz}{2\pi i} J_n^a(z)$ is

$$[J_n^{\hat{I}}, J_m^{\hat{J}}] = in\delta^{\hat{I}\hat{J}}\delta_{n+m,0}\partial y\,,$$

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- For finite Wilson line $X^{\hat{I}}(z) \to X'^{\hat{I}}(z) = X^{\hat{I}}(z) - A^{\hat{I}}Y(z)$

$$X'^{\hat{I}}(z)X'^{\hat{J}}(w) \sim -\left(\delta^{\hat{I}\hat{J}} + \frac{A^{\hat{I}}A^{\hat{J}}}{2R^2}\right)\log(z-w)\,,\ X'^{\hat{I}}(z)Y(w,\bar{w}) \sim \frac{A^{\hat{I}}}{2R^2}\log(z-w)$$

they obey the same algebra.